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RANKING GENERALIZED INTERVAL VALUED PENTAGONAL FUZZY NUMBERS TO SOLVE FUZZY TRANSPORTATION PROBLEM

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Abstract: In day today life, decision makers come across uncertain and incomplete information. Depending on the graphical representation, the suitable uncertain numbers are chosen to model the real life situation mathematically. The generalized interval valued pentagonal fuzzy numbers are much better in representing the imprecise information. The main objective of this paper is to justify the possibility of representing the vague information as generalized interval valued fuzzy numbers.The ranking technique average of the $\alpha-$cut of a generalized interval valued pentagonal fuzzy number [GIVPFN] is also proposed. The ranking formula is set up to defuzzify and solve the fuzzy transportation problem [GIVPFTP] whose costs are generalized interval valued pentagonal fuzzy numbers. This new defuzzification technique is validated by the comparative analysis.

Keywords and Phrases: Interval Valued Pentagonal Fuzzy Numbers, Generalized Interval Valued Pentagonal Fuzzy Numbers, Generalized Fuzzy Transportation Problem, and Ranking of Generalized Interval Valued Pentagonal Fuzzy Numbers.

2020 Mathematics Subject Classification: 03E72, 90B06.

1. Introduction

Zadeh L A [15] introduced the fuzzy set theory in 1965. Chang and Zadeh [5] introduced the concept of fuzzy numbers.The sets defined on the real line $\mathbb{R}$, known as fuzzy numbers. Fuzzy numbers are used to model imprecise quantities such as about three, close to three, below three, more or less three, nearly three and the like in a real-world manner. R E Bellman and L A Zadeh [1970] [3], Decision making in a fuzzy environment. Sambuc [12] extends fuzzy set theory to the concepts like interval-valued fuzzy sets in 1975. In 1986, Atanassov [2] introduced the intuitionistic fuzzy sets as a generalization of the theory of fuzzy sets. These notions are apt for dealing with different problems in situations involving uncertainty and vagueness. Interval-valued triangular fuzzy numbers, interval-valued trapezoidal fuzzy numbers and interval-valued pentagonal fuzzy numbers are introduced and being applied in the various decision-making problems. When the experts give their opinions in intervals, we need to accommodate their opinions without generalizing.

Ranking of fuzzy numbers plays a vibrant role in fuzzy arithmetic and fuzzy decision-making. An efficient parameter for ordering the fuzzy numbers is the ranking technique, which maps each fuzzy number into the real line, where a natural order survives. Ponnivalavan and Pathinathan [10, 11] introduced intuitionistic pentagonl fuzzy numbers with basic arithmetic operations and used the accuracy function as a ranking technique. Pathinathan T and Minj A [7, 9] explored the relation between pentagonal intuitionistic fuzzy number and interval-valued pentagonal fuzzy number by using the relationship between interval-valued fuzzy sets and intuitionistic fuzzy sets. Siji and Selva Kumari [14] also established a method for solving Network problem with pentagonal intuitionistic fuzzy numbers using accuracy function as ranking technique. Helen and Uma [6] presented a new arithmetic operation and ranking on pentagonal fuzzy numbers. Annie Christi and Kasthuri [1] achieved a solution for transportation problem with pentagonal intuitionistic fuzzy numbers using ranking technique and Russell's Method. Shumnugapriya S and Uthra G [13] discussed the centroid ranking technique for interval valued fuzzy numbers. Sankar Prasad Mondal *et.al.,* [8], proposed the solution of integral equations with pentagonal intuitionistic fuzzy numbers. Avishek Chakraborty *et.al.,* [4], discussed the pentagonal fuzzy numbers; Its different representations, properties, ranking, defuzzification and application in game problems.

Decision maker has to choose the right tool to optimize the practical problems. Modeling such problems is of foremost important. Mathematical model of generalized interval valued pentagonal fuzzy transportation problem [GIVPFTP]is demonstrated for the comfort of any decision-maker. The paper formulates the transportation problem with generalized interval valued pentagonal fuzzy numbers as costs to deal with uncertainty in transportation problem. The new ranking measure proposed in this paper proves to be efficient over the other fuzzy ranking existing techniques. The optimum solution of generalized interval valued pentagonal fuzzy transportation problem is found by applying the VAM and MODI method.

This paper is structured as follows: In Section 2, the basic definitions of fuzzy set and fuzzy numbers, the concepts of generalized interval valued pentagonal fuzzy numbers, Arithmetic addition of generalized interval valued pentagonal fuzzy numbers is reviewed. Section 3, exhibits the fundamental model of the fuzzy transportation problem and its mathematical formulation, the proposed algorithm, proposed ranking function, ordering, the numerical example proving the efficiency of proposed approach with comparative analysis. Finally, Section 4 reveals the validation of proposed approach, conclusion and future work.

2. Basic Definition

Definition 2.1. (Fuzzy Set) *A Fuzzy set $\widetilde{A}$ is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval $[0,1]$. (i.e.) $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)); x \in X\}$, here $\mu_{\widetilde{A}} : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\widetilde{A}$ and $\mu_{\widetilde{A}}(x)$ called the membership function value of $x \in X$ in the fuzzy set. These membership grades are often represented by real numbers ranging from $[0,1]$.*

Definition 2.2. (Fuzzy Number) *A fuzzy set $\widetilde{A}$, defined on the universal set of real numbers $\mathbb{R}$, is said to be a fuzzy number if its membership function has the following characteristics:*

- $\mu_{\widetilde{A}}(x) : R \to [0,1]$ *is continuous.*
- $\mu_{\widetilde{A}}(x) = 0$ *for all* $x \in (-\infty, a] \cup [d, \infty)$.
- $\mu_{\widetilde{A}}(x)$ *strictly increasing on* $[a,b]$ *and strictly decreasing on* $[c,d]$.
- $\mu_{\widetilde{A}}(x) = 1$ *for all* $x \in [b,c]$, *where* $a < b < c < d$.

Definition 2.3. (Generalized Fuzzy Number) *A fuzzy set $\widetilde{A}$, defined on the universal set of real numbers $\mathbb{R}$, is said to be generalized fuzzy number if its membership function has the following characteristics:*

- $\mu_{\widetilde{A}}(x) : R \to [0,\omega]$ *is continuous.*
- $\mu_{\widetilde{A}}(x) = 0$ *for all* $x \in (-\infty, a] \cup [d, \infty)$.
- $\mu_{\widetilde{A}}(x)$ *strictly increasing on* $[a,b]$ *and strictly decreasing on* $[c,d]$.

- $\mu_{\underset{A}{\sim}}(x) = \omega$ *for all* $x \in [b, c]$, *where* $0 < \omega \leq 1$.

Definition 2.4. (Interval Valued Fuzzy Set) *An Interval Valued Fuzzy set (IVFS)* $\tilde{A}$ *on* $\mathbb{R}$ *is defined by*
$\overset{\sim IV}{A} = \left[\{x, \left(\mu^U_{\underset{A}{\sim}IV}(x), \mu^L_{\underset{A}{\sim}IV}(x)\right)\} : x \in \mathbb{R}\right]$ *where* $x \in \mathbb{R}$ *and* $\mu^U_{\underset{A}{\sim}IV}(x)$ *maps* $\mathbb{R}$ *into* $[0, 1]$, $\mu^L_{\underset{A}{\sim}IV}(x)$ *maps* $\mathbb{R}$ *into* $[0, 1]$.

Definition 2.5. (Interval Valued Pentagonal Fuzzy Number) *An Interval valued pentagonal fuzzy number is written as* $\overset{\sim IV}{A} = \{(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5);$ $\omega^L_{\underset{A}{\sim}IV}, \omega^U_{\underset{A}{\sim}IV}\}$, *where* $b_1 \leq a_1 \leq b_2 \leq a_2 \leq b_3 \leq a_3 \leq a_4 \leq b_4 \leq a_5 \leq b_5$, $0 \leq k_1 \leq k_2 \leq \omega^L_{\underset{A}{\sim}IV} \leq \omega^U_{\underset{A}{\sim}IV}\} \leq 1$, *whose corresponding membership function is*

$$\mu^L_{\underset{A}{\sim}IV}(x) = \left\{\begin{array}{rc} 0, & x < a_1 \\ k_1 - \frac{k_1(x-a_2)}{a_1-a_2}, & a_1 \leq x \leq a_2 \\ \omega^L_{\underset{A}{\sim}IV} + \frac{(k_1-\omega^L_{\underset{A}{\sim}IV})(x-a_3)}{a_2-a_3}, & a_2 \leq x \leq a_3 \\ \omega^L_{\underset{A}{\sim}IV} + \frac{(k_1-\omega^L_{\underset{A}{\sim}IV})(x-a_3)}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ k_1 - \frac{k_1(x-a_4)}{a_5-a_4}, & a_4 \leq x \leq a_5 \\ 0, & x > a_5 \end{array}\right\}.$$

$$\mu^U_{\underset{A}{\sim}IV}(x) = \left\{\begin{array}{rc} 0, & x < b_1 \\ k_2 - \frac{k_2(x-b_2)}{b_1-b_2}, & b_1 \leq x \leq b_2 \\ \omega^U_{\underset{A}{\sim}IV} + \frac{(k_2-\omega^U_{\underset{A}{\sim}IV})(x-b_3)}{b_2-b_3}, & b_2 \leq x \leq b_3 \\ \omega^U_{\underset{A}{\sim}IV} + \frac{(k_2-\omega^U_{\underset{A}{\sim}IV})(x-b_3)}{b_4-b_3}, & b_3 \leq x \leq b_4 \\ k_2 - \frac{k_2(x-b_4)}{b_5-b_4}, & b_4 \leq x \leq b_5 \\ 0, & x > b_5 \end{array}\right\}.$$

Definition 2.6. (Arithmetic operations on Generalized Interval valued pentagonal fuzzy number)
Let $\overset{\sim IV}{A} = \{(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5); \omega^L_{\underset{A}{\sim}IV}, \omega^U_{\underset{A}{\sim}IV}\}$ *and*
$\overset{\sim IV}{B} = \{(p_1, p_2, p_3, p_4, p_5), (q_1, q_2, q_3, q_4, q_5); u^L_{\underset{A}{\sim}IV}, u^U_{\underset{A}{\sim}IV}\}$, *be two GIVPFNs. Then*

Addition:

$$\overset{\sim IV}{A} + \overset{\sim IV}{B} = \Big[\big[(a_1, a_2, a_3, a_4, a_5), \omega^L_{\overset{\sim IV}{A}}\big], \big[(b_1, b_2, b_3, b_4, b_5); \omega^U_{\overset{\sim IV}{A}}\big]\Big]$$
$$+ \Big[\big[(p_1, p_2, p_3, p_4, p_5), u^L_{\overset{\sim IV}{A}}\big], \big[(q_1, q_2, q_3, q_4, q_5); u^U_{\overset{\sim IV}{A}}\big]\Big]$$
$$= \Big[\big[(a_1 + p_1, a_2 + p_2, a_3 + p_3 + a_4 + p_4, a_5 + p_5), \omega\big], \big[(b_1 + q_1, b_2 + q_2, b_3 + q_3, b_4 + q_4, b_5 + q_5), u\big]\Big],$$

where $\omega = min\{\omega^L_{\overset{\sim IV}{A}}, \omega^U_{\overset{\sim IV}{A}}\}$ *and* $u = max\{u^L_{\overset{\sim IV}{A}}, u^U_{\overset{\sim IV}{A}}\}$

Difference:

$$\overset{\sim IV}{A} - \overset{\sim IV}{B} = \Big[\big[(a_1, a_2, a_3, a_4, a_5), \omega^L_{\overset{\sim IV}{A}}\big], \big[(b_1, b_2, b_3, b_4, b_5); \omega^U_{\overset{\sim IV}{A}}\big]\Big]$$
$$- \Big[\big[(p_1, p_2, p_3, p_4, p_5), u^L_{\overset{\sim IV}{A}}\big], \big[(q_1, q_2, q_3, q_4, q_5); u^U_{\overset{\sim IV}{A}}\big]\Big]$$
$$= \big[max(a_1 - p_5, 0), max(a_2 - p_4, 0), max(a_3 - p_3, 0), max(a_4 - p_2, 0), max(a_5 - p_1, 0), max(b_1 - q_5, 0), max(b_2 - q_4, 0), max(b_3 - q_3, 0), max(b_4 - q_2, 0), max(b_5 - q_1, 0), u\big],$$

where $\omega = min\{\omega^L_{\overset{\sim IV}{A}}, \omega^U_{\overset{\sim IV}{A}}\}$ *and* $u = max\{u^L_{\overset{\sim IV}{A}}, u^U_{\overset{\sim IV}{A}}\}$

Scalar Multiplication:

$$\lambda\overset{\sim IV}{A} = \lambda\big[(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5); \omega^L_{\overset{\sim IV}{A}}, \omega^U_{\overset{\sim IV}{A}}\big]$$
$$= \big[(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5), (\lambda b_1, \lambda b_2, \lambda b_3, \lambda b_4, \lambda b_5); \omega^L_{\overset{\sim IV}{A}}, \omega^U_{\overset{\sim IV}{A}}\big], \lambda \geq 0.$$

Definition 2.7. *Let* $\overset{\sim IV}{A}$ *and* $\overset{\sim IV}{B}$ *be two IVPFNs. The ranking of* $\overset{\sim IV}{A}$ *and* $\overset{\sim IV}{B}$ *by the* $\mathbb{R}(.)$ *on the set of IVPFN's is defined as follows:*

(i) $R(\overset{\sim IV}{A}) > R(\overset{\sim IV}{B})$ *iff* $\overset{\sim IV}{A} > \overset{\sim IV}{B}$

(ii) $R(\overset{\sim IV}{A}) < R(\overset{\sim IV}{B})$ *iff* $\overset{\sim IV}{A} < \overset{\sim IV}{B}$

(iii) $R(\overset{\sim IV}{A}) = R(\overset{\sim IV}{B})$ *iff* $\overset{\sim IV}{A} = \overset{\sim IV}{B}$

3. Solution of Fuzzy Transportation Problem

Mathematical model of generalized interval valued pentagonal fuzzy transportation problem is displayed for the decision-making purpose. In this section, the novel ranking technique [3.3] is proposed to defuzzify the generalized interval valued pentagonal fuzzy numbers. Further, the optimum solution of generalized interval valued pentagonal fuzzy transportation problem is obtained by applying the VAM and MODI method.

3.1. Generalized Interval Valued Pentagonal Fuzzy Transportation Model

Let us assume that there are m sources and n destinations. Let $\tilde{a}_i$ be the fuzzy supply at i, $\tilde{b}_j$ be the fuzzy demand at destination j, $\tilde{C}_{ij}^{IVP}$ be the unit fuzzy transportation cost from source i to destination j and $\tilde{X}_{ij}$ be the number of units shifted from source i to destination j. The fuzzy transportation problem can be mathematically expressed as

$$Minimize\ \ \tilde{z} = \Sigma_{i=1}^{m}\Sigma_{j=1}^{n}\tilde{C}_{ij}^{IVP}\tilde{X}_{ij}$$

Subject to the constraints $\Sigma_{j=1}^{n}\tilde{X}_{ij} = \tilde{a}_i,\ i = 1, 2, 3, \cdots, m, \Sigma_{i=1}^{m}\tilde{X}_{ij} = \tilde{b}_j,\ j = 1, 2, 3, \cdots, n$ and $\tilde{X}_{ij} \geq 0$ for all i and j.

A fuzzy transportation problem is said to be balanced if the total fuzzy supply from all sources equal to the total fuzzy demand in all destination $\Sigma_{i=1}^{m}\tilde{a}_i = \Sigma_{j=1}^{n}\tilde{b}_j$, otherwise it is called unbalanced.

The generalized interval valued pentagonal fuzzy transportation problem [GIVP FTP] can be represented in the form of $m \times n$ fuzzy cost table, with cost presented as $\tilde{C}_{ij}^{IVP}$

	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	Supply
O_1	$\tilde{C}_{11}^{IVP}$	$\tilde{C}_{12}^{IVP}$	$\cdots$	$\tilde{C}_{1j}^{IVP}$	$\cdots$	$\tilde{C}_{1n}^{IVP}$	$\tilde{a}_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
O_i	$\tilde{C}_{i1}^{IVP}$	$\tilde{C}_{i2}^{IVP}$	$\cdots$	$\tilde{C}_{ij}^{IVP}$	$\cdots$	$\tilde{C}_{im}^{IVP}$	$\tilde{a}_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
O_m	$\tilde{C}_{m1}^{IVP}$	$\tilde{C}_{m2}^{IVP}$	$\cdots$	$\tilde{C}_{mj}^{IVP}$	$\cdots$	$\tilde{C}_{mn}^{IVP}$	$\tilde{a}_m$
Demand	$\tilde{b}_1$	$\tilde{b}_2$	$\cdots$	$\tilde{b}_j$	$\cdots$	$\tilde{b}_n$	

Table 1: Fuzzy Transportation Model

Here the costs $\tilde{C}_{ij}^{IVP}$ is generalized interval valued pentagonal fuzzy numbers denoted by $\tilde{C}_{ij}^{IVP} = \left([C_p^L, C_q^U], [C_l^L, C_m^U]\right)$. The aim is to minimize the fuzzy cost incurred in transportation effectively.

3.2. Justification of the Proposed Approach

The proposed approach of solving generalized interval valued pentagonal fuzzy transportation problem is justified by the following procedure to authorize the efficiency and computational ease of the approach.

1. Ranking and ordering,
2. Defuzzification,
3. VAM and MODI method and
4. Validation

3.3. Proposed Ranking Technique of GIVPFN

Let $\overset{\sim IV}{A} = \{(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5); \omega^L_{\underset{A}{\sim IV}}, \omega^U_{\underset{A}{\sim IV}}\}$, where $b_1 \leq a_1 \leq b_2 \leq a_2 \leq b_3 \leq a_3 \leq a_4 \leq b_4 \leq a_5 \leq b_5,\ 0 \leq k_1 \leq k_2 \leq \omega^L_{\underset{A}{\sim IV}} \leq \omega^U_{\underset{A}{\sim IV}}\} \leq 1$, be the generalized interval valued pentagonal fuzzy number.

Average of the α-cut of a GIVPFN:

$A\big(\mu^L_{\underset{A}{\sim IV}}(\alpha_1)\big) = \frac{1}{16}\Bigg[\big(1-\frac{\alpha_1}{k_1}\big)(a_1+a_5-a_2-a_4)+\frac{(\alpha_1-\omega^L_{\underset{A}{\sim IV}})}{(k_1-\omega^L_{\underset{A}{\sim IV}})}(a_2-2a_3+a_4)+a_2+2a_3+a_4\Bigg]$

and $A\big(\mu^U_{\underset{A}{\sim IV}}(\alpha_2)\big) = \frac{1}{16}\Bigg[\big(1-\frac{\alpha_2}{k_2}\big)(b_1+b_5-b_2-b_4)+\frac{(\alpha_2-\omega^U_{\underset{A}{\sim IV}})}{(k_2-\omega^U_{\underset{A}{\sim IV}})}(b_2-2b_3+b_4)+b_2+2b_3+b_4\Bigg]$

The proposed ranking function is defined as follows:

$$R(\overset{\sim IV}{A}) = \frac{A\Big(\mu^L_{\underset{A}{\sim IV}}(\alpha_1)\Big)\omega^L_{\underset{A}{\sim IV}} + A\Big(\mu^L_{\underset{A}{\sim IV}}(\alpha_2)\Big)\omega^U_{\underset{A}{\sim IV}}}{2}$$

3.4. Ordering Technique of GIVPFN

Case I: The ordering $\geq$ and $\leq$ between any two IVPFN s $\overset{\sim IV}{A}$ and $\overset{\sim IV}{B}$ are defined as follows:

(i) $\overset{\sim IV}{A} \geq \overset{\sim IV}{B}$ iff $\overset{\sim IV}{A} > \overset{\sim IV}{B}$ or $\overset{\sim IV}{A} = \overset{\sim IV}{B}$ and

(ii) $\overset{\sim IV}{A} \leq \overset{\sim IV}{B}$ iff $\overset{\sim IV}{A} < \overset{\sim IV}{B}$ or $\overset{\sim IV}{A} = \overset{\sim IV}{B}$

Case II: Let $\{\overset{\sim IV}{A_i}, i = 1, 2, \cdots, n\}$ be a set of IVPFN s. If $R(\overset{\sim IV}{A_k}) \leq R(\overset{\sim IV}{A_i})$ for all i, then the IVPFN $\overset{\sim IV}{A_k}$ is the minimum of $\{\overset{\sim IV}{A_k}, i = 1, 2, \cdots, n\}$.

Case III: Let $\{\overset{\sim IV}{A_i}, i = 1, 2, \cdots, n\}$ be a set of IVPFN s. If $R(\overset{\sim IV}{A_k}) \geq R(\overset{\sim IV}{A_i})$ for all i, then the IVPFN $\overset{\sim IV}{A_k}$ is the minimum of $\{\overset{\sim IV}{A_i}, i = 1, 2, \cdots, n\}$.

3.5. Procedure to Solve the Generalized Interval Valued Pentagonal Fuzzy Transportation Problem

In this section the flow chart for ranking measure and algorithm to solve GIVPFTP is displayed for the ease of decision-maker. The proposed ranking measure [3.3] is applied to order and defuzzify the generalized interval valued pentagonal fuzzy numbers, obtained from a real time source. Further, the optimum solution of generalized interval valued pentagonal fuzzy transportation problem [GIVPFTP] is obtained by using the VAM and MODI method.

3.5.1. Proposed Approach to Solve GIVPFTP

Step 1: Find the rank of each cell $\overset{\sim IVP}{C_{ij}}$ of the chosen generalized interval valued pentagonal fuzzy cost matrix by using the ranking function as mentioned in section 3. If the number of sources is equal to the number of destinations, go to step 3. If the number of sources is not equal to the number of destinations go to step 2.

Step 2: Introduce dummy rows or dummy columns with zero fuzzy costs to form a balanced one.

Step 3: Proceed by the VAM method to find the initial basic feasible solution and if $m + n - 1$ = number of allocations, then proceeds by MODI method to obtain the optimal solution.

Step 4: Add the optimal fuzzy cost using fuzzy addition mentioned in section 2, to optimize the cost.

3.5.2. Flow Chart

The flow chart of the proposed ranking measure is shown below:

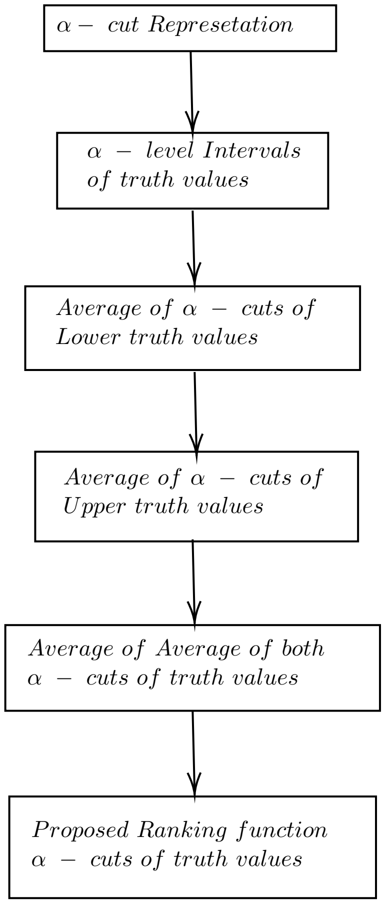

Figure 1: Flow Chart of Defuzzification Function

3.6. Numerical Illustration

This section proposes an optimal solution of a fuzzy transportation problem whose costs are generalized interval valued pentagonal fuzzy numbers. The proposed new defuzzification technique followed by VAM and MODI method yields the minimum cost of transportation. The approach is justified through the comparative analysis [Data obtained from Shree Vengateswaraa Pharmaceuticals-Chennai, Tamil Nadu]

Consider a fuzzy transportation problem with three origins O_1, O_2, O_3 and three destinations D_1, D_2, D_3, whose costs (in lakhs) are considered to be generalized interval valued pentagonal fuzzy numbers. The problem is to find the optimal transportation in an efficient way. (Units of supply and demand -1000 Boxes).

	D_1	D_2	D_3	Supply
O_1	(2,4,5,6,8;0.8) (1,3,5,7,9;1.0)	(1,2,3,4,5;0.7) (0,1,3,5,6;1.0)	(2.5,4.5,5.5,6.5,8.5;0.7) (2,4,5.5,7,9;0.9)	0.25
O_2	(1.5,2.5,3.5,4.5,5.5;0.7) (1,2,3.5,5,6;0.9)	(2,4,6,8,10;0.7) (1,3,6,9,11;0.8)	(1,3,5,7,9;0.8) (0,2,5,8,10;1.0)	0.75
O_3	(1,3,5,7,9;0.8) (0,2,5,8,10;0.9)	(2.5,4.5,5.5,6.5,8.5;0.7) (1.5,3.5,5.5,7.5,9.5;1.0)	(2,3,4,5,7;0.7) (1,2,4,6,8;0.8)	0.5
Demand	0.25	0.5	0.75	

Table 2: Generalized Interval Valued Pentagonal Fuzzy Cost Table

Solution: The given generalized interval valued pentagonal fuzzycost table is balanced one.
Applying the rule of defuzzification by weighted average of alpha $(\alpha)-$cut technique, the generalized interval valued pentagonal fuzzy cost is defuzzified to crisp data.

	D_1	D_2	D_3	Supply
O_1	1.125	0.6375	1.1	0.25
O_2	0.7	1.125	1.125	0.75
O_3	1.0625	1.1688	0.7735	0.5
Demand	0.25	0.5	0.75	

Table 3: Defuzzified Transportation Table

Proceeding by VAM method, $m+n-1 =$ the number of allocations, the initial basic feasible solution is non- degenerate and optimal. Proceeding by MODI method, the optimum solution is:

	D_1	D_2	D_3	Supply
O_1	1.125	0.25 0.6375	1.1	0.25
O_2	0.25 0.7	0.25 1.125	0.25 1.125	0.75
O_3	1.0625	1.1688	0.5 0.7735	0.5
Demand	0.25	0.5	0.75	

Table 4: Optimum Allocation Table

Therefore, the transportation cost = $(0.6375 * 0.25) + (0.7 * 0.25) + (1.125 * 0.25) + (1.125 * 0.25) + (0.7735 * 0.5) = 1.2836$.

3.7. Validation

In the numerical illustration [3.6], the generalized interval valued pentagonal fuzzy numbers are defuzzified by the proposed ranking technique [3.3]. The traditional VAM and MODI method is applied to solve the defuzzified cost matrix. The proposed approach is validated by the comparative analysis [3.4].

Defuzzification Technique / Interval Valued Pentagonal Fuzzy Number	(2,4,5,6,8;0.8) (1,3,5,7,9;1.0)	(2.5,4.5,5.5,6.5,8.5;0.7) (1.5,3.5,5.5,7.5,9.5;1.0)
Proposed Method	1.125	1.75
Sign Distance [7]	1.1688	1.888

Table 5: Comparative Analysis

4. Inference

This paper proposes an optimal solution of a fuzzy transportation problem whose costs are taken as generalized interval valued pentagonal fuzzy numbers. The proposed new defuzzification technique yields minimum cost, compared to other methods in dealing such complex fuzzy transportation problems. The comparison table proves the effectiveness of the proposed approach in minimizing cost and time as compared to the other ranking methods.

As a future extension, the proposed algorithm may be used to solve, assignment problems using linguistic variables, transportation problems involving linguistic expressions, higher order interval valued transportation problems.

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